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Strawsonian presuppositions and logical entailment

Abstract. We formalize and investigate by means of logical entailment two of Strawson's notions of presupposition: Strawsonian presupposition and presupposition via negation. We develope the theory of bi-matrices which are a formal tool to investigate Strawsonian presuppositions. We prove that any class of presuppositional bi-matrices determine the Strawsonian presupposition operator which has only tautological presuppositions. We also prove that virtually all logical consequences determine a notion of presupposition via negation which admits only tautological presuppositions[1].

Keywords. Logical entailment, presupposition, logical matrices, bi-matrices.

Introduction.

In 1892 [1952 p. 69] Frege systematically investigated the notion of presupposition. According to Frege:

"If anything is asserted there is always an obvious presupposition the the simple or compound proper names used have a reference. If one therefore asserts 'Kepler died in misery', there is presupposition that the name 'Kepler' designates something."

Since Frege's seminal paper a number of approaches to dealing with the phenomenon of presupposition have been proposed. It is not the aim of this paper to analyze the phenomenon of presupposition. For review of approaches to presupposition we refer the reader to Levinson [83]. A review of more recent results in the subject can be find in Beaver [97]. The aim of

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present the paper is to analyze from logical point of view just one approach to the concept of presupposition – a concept known in the literature as Strawson's notion of presupposition. According to the original proposal presented by Strawson in [49] (p. 175): "a statement S presupposes a statement S' if and only if the truth of S' is a precondition of the truth-or-falsity of S". The *Strawsonian presupposition* is usually considered in the literature in the following form: P presupposes Q if and only if Q is true provided P is true or P is false.

Under very weak additional assumptions the definition above is equivalent to the following concept of *presupposition via negation* - P presupposes Q if and only if P entails Q and $\neg P$ entails Q.

David Beaver [97] (p. 948) formulates *three-valued Strawsonian presupposition* by means of three-valued possible worlds semantics: P presupposes Q if and only if for all possible worlds w if P in the world w is true or false then Q is true in the world w. Any sentence P may take in a world w any of the three semantic values: true, false and undefined.

Strawsonian presupposition is based on two other notions: a logical entailment and logical value of falsity, while the *presupposition via negation* depends on a logical entailment and the negation connective. In this paper we formalize in a general way both of the above definitions. In the first section we introduce the main notions concerning logical consequence operations and logical matrices. In the second section we develop the theory of bi-matrices and operations determined by them. In the third section we use bi-matrices as a tool to formalize Strawsonian presupposition. We prove that for broad classes K of bi-matrices the notion of presupposition determined by this class has the unwanted property that all the sentences have only tautological presuppositions (theorem 3.4 and 3.5). The class K contains, among others, bi-matrices determining all the logics expressed in the language of classical logic and contained in the classical logic (3.2, 3.3).

The rest of third section concerns presupposition via negation. For a given logical consequence operation C defined in a sentential language with a negation $\neg$ we say that P C-presupposes Q if and only if $Q \in C(P)$ and $Q \in C(\neg P)$. We introduce the notion of a sub-classical logic. A logic is sub-classical if and only if its reduct to classical connectives is weaker (as a consequence operation) than the classical logic. For example, all the logics obtained from the classical logic by means of adding new operators are sub-classical. We prove that for any sub-classical consequence operation any sentence of classical logic has only tautological properties (3.9).

The notions of Strawsonian presupposition and that of presupposition via negation have different but overlapping domains. It is easy to prove that these notions are equivalent in the common part of their domains.

1 Logical consequence operation

By a *sentential language* we shall mean an absolutely free algebra $S = (S, f_1, ..., f_n)$ freely generated by an infinite, countable set $Var(S)$ of *sentential variables*. The operations $f_1, ..., f_n$ of the algebra S will be called connectives. We shall assume that the language has a finite number of connectives. The elements of S will be called *sentences* or *formulas*.

Let us assume that the connective f_i is an operation with $u(i)$ arguments. The set of sentences of the language S may be described equivalently by the following recursive definition:

(i) Every sentential variable $p \in Var(S)$ is a sentence, i.e. $p \in S$.

(ii) If $\alpha_1, ..., \alpha_{u(k)}$ are sentences, then $f_k(\alpha_1, ..., \alpha_{u(k)})$ is a sentence, i.e. $f_k(\alpha_1, ..., \alpha_{u(k)}) \in S$.

The set of all sentential variables occurring in α will be denoted by $Var(\alpha)$. For any $X \subseteq S$, by $Var(X)$ we denote the set $\bigcup\{Var(\alpha) : \alpha \in X\}$. By $X(p_1, ..., p_n)$ (or, respectively, $\alpha(p_1, ..., p_n)$) we shall mean, that $\{p_1, ..., p_n\} \subseteq Var(X)$ (respectively $\{p_1, ..., p_n\} \subseteq Var(\alpha)$).

By a *substitution* in S we mean any endomorphism $e \in End(S)$, i.e. any homomorphism of the language S onto itself. From the definition of the language S as a free algebra it follows that any function $\overline{e} : Var(S) \longrightarrow Var(S)$ can be extended to an endomorphism, i.e. a substitution $e : S \longrightarrow S$ in a unique way. Thus every substitution is determined uniquely by its values on sentential variables i.e. every function $\overline{e} : Var(S) \longrightarrow S$ determines exactly one substitution. This lets us accept the following convention: Let $\alpha(p_1, ..., p_n), \beta_1, \ldots, \beta_n$ be formulas, then $\alpha(\beta_1/p_1, \ldots, \beta_n/p_n)$ is the formula obtained from α by substituting every occurrence of the variable p_i by the sentence β_i for $i \in \{1, \ldots, n\}$. We will often shorten $\alpha(\beta_1/p_1, \ldots, \beta_n/p_n)$ to $\alpha(\beta_1, \ldots, \beta_n)$.

If e is a substitution in S such, that for $i \in \{1, \dots, n\}$ we have $e(p_i) = \beta_i$, then $\alpha(\beta_1/p_1, ..., \beta_n/p_n) = e(\alpha)$. For any set of sentences $X(p_1, \dots, p_n)$, $X(\alpha_1, ..., \alpha_n)$ denotes the set $\{\alpha(\alpha_1/p_1, ..., \alpha_n/p_n) : \alpha \in X\}$.

We will say that a set X is *closed under substitutions* if and only if for any substitution e $e(X) \subseteq X$.

Let a language S be given. A function:

$$C : S \supseteq X \longmapsto C(X) \subseteq S,$$

satisfying conditions:

$X \subseteq C(X)$,

if $X \subseteq Y$, then $C(X) \subseteq C(Y)$,

$CC(X) = C(X)$

will be called a *consequence operation*. If, in addition, C satisfies the following principle of structurality:

$e(C(X)) \subseteq C(e(X))$

for any substitution e in S and any set $X \subseteq S$, then C will be called a *structural* consequence operation. The consequence operation C will be called *finitary* if for any set of sentences $X \subseteq S$ and a sentence $\alpha \in S$, from the fact that $\alpha \in C(X)$ it follows that there exists a finite subset Y of the set X such, that $\alpha \in C(Y)$. A consequence operation that is both finitary and structural will be called *standard*.

Let S be a sentential language and C — a structural consequence operation on S. The pair (S, C) will be called a *logic*. If a structural consequence operation C is given, then it is clear what its language is. For this reason we will identify the logic (S, C) with its consequence operation C and hence the structural operation C will be often called a logic.

When X is a finite set $X = \{\alpha_1, ..., \alpha_n\}$, then instead of $C(X)$ we shall often write $C(\alpha_1, ..., \alpha_n)$, also instead of $C(X \cup \{\alpha\})$ we shall write $C(X, \alpha)$.

We shall call the set of sentences X of the language S a C-*theory* or a *theory of an operation* C if and only if $X = C(X)$. The set of all C-theories will be denoted by $Th(C)$.

A logic C' is called *stronger* than a logic C (we write $C \leq C'$) if for any set of sentences $X \subseteq S$, we have $C(X) \subseteq C'(X)$. A logic C is called *weaker* than a logic C') if and only if C' stronger than a logic C. A relation $\leq$ defined in this way on the set of all logics stroner than a given logic C is a relation of partial order. Moreover, a family of all strengthenings of a logic C is a complete lattice relative to this order. If $C \leq C'$, then we shall often say that C is weaker than C', and C' is stronger than C.

THEOREM 1.1. *Let C_1 and C_2 be consequence operations in a given language S. The following conditions are equivalent:*

(i) $C_1 \leq C_2$.

(ii) $Th(C_2) \subseteq Th(C_1)$.

Proof: $(i) \Rightarrow (ii)$. Suppose that $C_1 \leq C_2$. Let $X \in Th(C_2)$, then $X \subseteq C_1(X) \subseteq C_2(X) = X$. Then $X = C_1(X)$ and hence $X \in Th(C_1)$.
$(ii) \Rightarrow (i)$. If $Th(C_2) \subseteq Th(C_1)$, then $C_1C_2(X) = C_2(X)$, so $C_1(X) \subseteq C_1C_2(X) = C_2(X)$ and, consequently, $C_1(X) \subseteq C_2(X)$, for any $X \subseteq S$. □

Every consequence operation is uniquely determined by its theories, since we have:

THEOREM 1.2. *If C_1 and C_2 are consequence operations, then $C_1 = C_2$ if and only if $Th(C_1) = Th(C_2)$.*

Proof. From left to right the proof is obvious. Let us suppose that $C_1 \neq C_2$, then there exist $X \subseteq S$ and $\alpha \in S$ such, that $\alpha \in C_1(X)$ and $\alpha \notin C_2(X)$ or $\alpha \notin C_1(X)$ and $\alpha \in C_2(X)$. Let, for example, $\alpha \in C_1(X)$ and $\alpha \notin C_2(X)$, then the set $C_2(X)$ is a C_2-theory.

Suppose that $C_2(X)$ is a C_1-theory, hence $\alpha \in C_1C_2(X) = C_2(X)$, then $\alpha \in C_2(X)$ what contradicts to our assumption. Hence $Th(C_1) = Th(C_2)$. □

A logical matrix is a pair $M = (A, D)$, where A is an abstract algebra and D - a subset of the set A. The algebra A is called an *algebra of a matrix* M, the set D will be called a *set of distinguished elements*.

Two matrices $M = (A, D)$ and $N = (B, E)$ are called *similar* if the algebras A and B are of the same type. Let S be a sentential language. We shall call a matrix $M = (A, D)$ a *matrix for the language* S if the algebras A and S are of the same type. Any class $\mathbf{K}$ of matrices for a language S we shall call a *matrix semantic* for S. In the remaining part of this work by a class of matrices we shall always mean a class of matrices that are of the same type. If $M = (A, D)$ is a matrix for S, then the elements of the set $Hom(S, A)$ of homomorphisms of the language S into the algebra A will be called *valuations* of S into A.

Every semantics $\mathbf{K}$ for S determines a function:

$$Cn_{\mathbf{K}} : S \supseteq X \longmapsto Cn_{\mathbf{K}}(X) \subseteq S$$

defined for any set of sentences $X \subseteq S$ and a sentence $\alpha \in S$ in the following way: $\alpha \in Cn_{\mathbf{K}}(X)$ if and only if for every matrix $M = (A, D) \in \mathbf{K}$ and every valuation $v \in Hom(S, A)$, if $v(X) \subseteq D$, then $v(\alpha) \in D$.

It is easy to prove that the function $Cn_{\mathbf{K}}$ is a structural consequence operation on S.

We say that a logic C is *strongly complete* relative to a semantics $\mathbf{K}$ for S, if $C = Cn_{\mathbf{K}}$. A semantics $\mathbf{K}$ is *strongly adequate* for a logic C if C is strongly complete relative to $\mathbf{K}$.

Let (S, C) be a logic. For any $X \subseteq S$ a matrix: $L_X = (S, C(X))$ is called a *Lindenbaum matrix* for C. A class of matrices $\mathbf{L}_C = \{L_X : X \subseteq L\}$ will be called a *Lindenbaum bundle* for C. One can prove that every logic is strongly complete relative to a Lindenbaum bundle $\mathbf{L}_C$.

Let C be a logic. We shall call a matrix M a *C-matrix*, if $C \leq Cn_M$. A class of all C-matrices will be denoted by $Mod(C)$. Of course $\mathbf{L}_C \subseteq Mod(C)$. Every logic C is strongly complete relative to a class $Mod(C)$. There exists a one–to–one correspondence between classes $Mod(C)$ and logics.

For the theory of logical matrices we refer the reader to R.Wojcicki [87], J.Malinowski [89] and J.Czelakowski [01]. Some results and notions concerning logical matrices will be generalized in the next section. In this section we will present only some property of sub-matrices and matrix congruences which will be used in this paper.

Given a logical matrix $M = (A, D)$, a binary relation $\equiv$ on A will be called a *matrix congruence* on M if and only if it is a congruence on A which does not paste together designated elements with non-designated ones. More precisely: $\equiv$ is an equivalence relation on A which agrees with the operations on A and moreover $a \equiv b$ and $a \in D$ if and only if $b \in D$. For a given logical matrix $M = (A, D)$, and matrix congruence $\equiv$ on A we can define the new *factor matrix* $M_{\equiv}$. Its algebra and its set of designated elements consist of classes of equivalence of the elements of A with respect to $\equiv$. Thus, $M_{\equiv} = (A_{\equiv}, D_{\equiv})$.

Let $M = (A, D)$ and $N = (B, E)$ be similar matrices. The matrix M will be called a *submatrix* of the matrix N, in symbols $M \subseteq N$, if A is a subalgebra of the algebra B and $D = A \cap E$.

THEOREM 1.3. *Let M, N be matrices for a language S. Let $\equiv$ be a matrix congruence on M, then*

a) if M is a submatrix of N, then $Cn_N \leq Cn_M$,

b) $C_M = C_{M_{\equiv}}$.

Proof. a) Let $M = (A_1, D_1)$ be a sub-matrix of the matrix $N = (A_2, D_2)$. Suppose that $\alpha \in Cn_N(X)$. Then for any valuation v of S into A_2 such that $v(X) \subseteq D_2$ we have $v(\alpha) \in D_2$. We have to prove that for any valuation $\overline{v}$ of S into A_1 such that $\overline{v}(X) \subseteq D_1$ we have $\overline{v}(\alpha) \in D_1$. Let id denotes the identical embedding of A_1 into A_2. Given any valuation $\overline{v}$ of S into A_1 such that $\overline{v}(X) \subseteq D_1$, then $id \circ \overline{v}$ is a valuation of S into A_2 such that $id \circ \overline{v}(X) \subseteq D_2 \cap A_1 = D_1$: Hence $\overline{v}(\alpha) \subseteq D_2 \cap A_1 = D_1$.

b) Let k denote the canonical function $A \ni a \longmapsto [a] \in A/\equiv$. By the definition of matrix congruence we have that $a \in D$ if and only if $k(a) = [a] \in D$. Moreover for any valuation $\overline{v} : S \longrightarrow A/\equiv$ there exists a valuation $v : S \longrightarrow A$ such, that $\overline{v}(P) = k(v(P))$.

Suppose that $P \in C_M(X)$, then for any valuation $v : S \longrightarrow A$ such, that $v(X) \subseteq D$ we have $v(P) \in D$. We will show that $P \in C_{M/\equiv}(X)$. Let $\overline{v} : S \longrightarrow A/\equiv$ denote any valuation such, that $\overline{v}(X) \subseteq D/\equiv$. There exists a valuation $v : S \longrightarrow A$ such, that $\overline{v}(P) = k(v(P))$. Obviously we have $v(X) \subseteq k^{-1}(\overline{v}(X)) \subseteq D$. From the assumption $v(P) \in D$ we have $\overline{v}(P) \in D/\equiv$.

Suppose that $P \in C_{M/\equiv}(X)$, then for any valuation $\overline{v} : S \longrightarrow A/\equiv$ such, that $\overline{v}(X) \subseteq D/\equiv$ we have $\overline{v}(P) \in D/\equiv$. We will show that $P \in C_M(X)$. Let $v : S \longrightarrow A$ denote any valuation such, that $v(X) \subseteq D$. We obviously have $\overline{v}(X) = k(v(X)) \subseteq D/\equiv$. From the assumption, $\overline{v}(P) \in D/\equiv$. As $a \in D$ iff $k(a) = [a] \in D$, hence $v(P) \in D$ $\square$.

2 Operations and bi-matrices.

By an *operation* we mean any function of the form:

$$F : S \supseteq X \longmapsto F(X) \subseteq S,$$

An operation satisfying the condition $X \subseteq F(X)$ will be called inclusive. An operation satisfying the condition if $X \subseteq Y$, then $F(X) \subseteq F(Y)$ will be called monotonic. An operation will be called idempotent if and only if it satisfies the condition $FF(X) = F(X)$. An operation is called structural if and only if it satisfies the condition: if $P \in F(X)$ then any substitution of P belong to $F(Y)$, where Y is a set all respective substitutions of sentences from the set X.

By a *bi-matrix* we mean a triple $M = (A, D, E)$ where A is an abstract algebra, D and E are the subsets of A – two sets of designated elements of M. We will say that two bi-matrices are similar if and only if their underlaying algebras are of the same type (i.e. have the same operations). By a *valuation* of S in a bi-matrix M we mean any homomorphism of the language S into the algebra A of the matrix M.

Every class of bi-matrices $\mathbf{K}$ determines a function:

$$Fn_{\mathbf{K}} : S \supseteq X \longmapsto Fn_{\mathbf{K}}(X) \subseteq S$$

defined in the following way: $P \in Fn_{\mathbf{K}}(X)$ if and only if for every bi-matrix $M = (A, D, E) \in \mathbf{K}$ and every valuation v, if $v(X) \subseteq D$, then $v(P) \in E$.

It is easy to check that $Fn_{\mathbf{K}}$ is a structural, monotonic operation.

We say that an operation F is *strongly complete* relative to a class of bi-matrices $\mathbf{K}$ for S, if $F = Fn_{\mathbf{K}}$. A class $\mathbf{K}$ of bi-matrices is *strongly adequate* for an operation F if F is strongly complete relative to $\mathbf{K}$.

Let F be an operation. For any $X \subseteq S$ a bi-matrix $L_X = (S, X, F(X))$ will be called a *Lindenbaum bi-matrix* for F. A class of bi-matrices $\mathbf{L}_F = \{L_X : X \subseteq L\}$ will be called a *Lindenbaum bi-bundle* for F.

Theorem 2.1. *Any structural monotonic operation F is strongly complete relative to the Lindenbaum bi-bundle $\mathbf{L}_F$.*

Proof. Given an operation F. Suppose that F is structural and monotonic. We will prove that $F = Fn_{\mathbf{K}}$ for

$$\mathbf{K} = \{M : M = (S, X, F(X)), X \subseteq S\}$$

Suppose that $P \in F(X)$. Given any $M = (S, Y, F(Y)) \in \mathbf{K}$ and any valuation $v : S \longmapsto S$ such, that $v(X) \subseteq Y$ then by monotonicity and structurality we have $v(P) \in v(F(X)) \subseteq F(v(X)) \subseteq F(Y)$.

Now suppose that $P \notin F(X)$. Lets consider the matrix $(S, X, F(X))$ and the identity function id as a valuation, then obviously $id(X) \subseteq X$ but $id(P) \notin F(X)$, and hence $P \notin Fn_{\mathbf{K}}(X)$. □

THEOREM 2.2. *Given a bi-matrix $M = (A, D, E)$.*
a) If $D \subseteq E$ then Fn_M is inclusive
b) If $E \subseteq D$ then Fn_M is idempotent
c) If $D = E$ then Fn_M is a structural consequence operation.

Proof. a) Lets assume that $D \subseteq E$, then for any valuation $v : S \longmapsto A$ such, that $v(X) \subseteq D$ we have $v(X) \subseteq E$. Hence $X \subseteq Fn_M(X)$.
b) Assume that $E \subseteq D$. Lets note that for any valuation $v : S \longmapsto A$ such, that $v(X) \subseteq D$ we have $v(Fn_M(X)) \subseteq E \subseteq D$.

Suppose that $P \notin Fn_M(X)$, then there exists a valuation $v : S \longmapsto A$ such, that $v(X) \subseteq D$ and $P \notin E$. From the remark above we have $v(Fn_M(X)) \subseteq D$. Hence $P \notin Fn_M Fn_M(X)$.
c) is an immediate consequence of a) and b). □

The notion of bi-matrix can serve as a tool not only for the description of the notion of logical entailment but also for the investigation other notions. One of them is the operator of presupposition. In the next section we will investigate it in detail. We shall consider it here just as an illustration of the notion of bi-matrix.

The operation **Pr** of presupposition is defined by means of the bi-matrix $Bm(\mathbf{sk3}) = (\mathbf{sk3}, \{0, 1\}, \{1\})$, where **sk3** denotes the strong three-valued Kleene algebra with operations of $\rightarrow$ $\wedge$, $\vee$ and $\neg$ defined in the following way:

sk3

$\vee$	0	1/2	1
0	0	1/2	1
1/2	1/2	1/2	1
1	1	1	1

$\wedge$	0	1/2	1
0	0	0	0
1/2	0	1/2	1/2
1	0	1/2	1

$\rightarrow$	0	1/2	1	$\neg$
0	1	1	1	1
1/2	1/2	1/2	1	1/2
1	0	1/2	1	0

Thus $P \in \mathbf{Pr}(X) = Fn_{Bm(\mathbf{sk3})}(X)$ if and only if for any bi-valuation v such, that $v(X) \subseteq \{0, 1\}$ we have $v(P) = 1$.

One can interpret the operation **Pr** in the following way: $P \in \mathbf{Pr}(X)$ means that P is true provided all the sentences from the set X have the classical logical value (i.e. are true or are false). Such a meaning perfectly mirrors Strawson's [49] approach to presupposition according to which a sentence P presupposes Q if and only if Q is true provided P is true or false. We leave the discussion of presuppositions to the last section.

The notion of operation can be obviously considered as a generalization of logical consequence operation and the notion of matrix as a generalization of logical matrix. Most of the notions introduced above for the consequence operation can be generalized for operations.

Given an operation F, we shall call the set of sentences X of the language S a *F-closed set* if and only if $X = F(X)$. The set of all F-closed sets will be denoted by $Th(F)$.

An operation F' is called a *stronger* than an operation F (we write $F \leq F'$) if for any set of sentences $X \subseteq S$, we have $F(X) \subseteq F'(X)$. A relation $\leq$ defined in this way on the set of all operations being stronger than a given operation F is a relation of partial order. Moreover, a family of all operation being stronger than a given operation F is a complete lattice relative to this order.

Of course in general the operations are not determined by its closed sets. Neither theorem (1.1) nor theorem (1.2) can be generalized for any operation. It is however easy to check that in the proof of (1.2) the condition of idempotency of C is not used. Then we have:

THEOREM 2.3. *If F_1 and F_2 are inclusive and monotonic operations then $F_1 = F_2$ if and only if $Th(F_1) = Th(F_2)$.* □

Suppose that F is structural and monotonic operation. We shall call a bi-matrix M a *F-bi-matrix*, if $F \leq Fn_M$. A class of all F-bi-matrices will be denoted by $Bimod(F)$.

Of course any Lindenbaum bi-matrix for F is F-bi-matrix: $\mathbf{L}_F \subseteq Bimod(F)$. So, from theorem (2.1) we can conclude that:

COROLLARY 2.4. *Every structural and monotonic operation F is strongly complete relative to a class $Bimod(F)$. At the same time the class $Bimod(F)$ is the greatest (in the sense of inclusion) class of bi-matrices with this property. Moreover, given two monotonic structural operations F_1 and F_2, for $F_1 = F_2$ it is necessary and sufficient that $Bimod(F_1) = Bimod(F_2)$.*

Proof. First statement is an obvious consequence of theorem (2.1). We will prove the second part. The proof from left to right is obvious. So let us suppose that $F_1 \neq F_2$. Then there exists a set $X \subseteq S$ and $\alpha \in S$ such, that

$\alpha \notin F_1(X)$ and $\alpha \in F_2(X)$. Surely, $(S, X, F_1(X)) \in Bimod(F_1)$, however $(S, X, F_1(X)) \notin Bimod(F_2)$. In fact, if $(S, X, F_1(X)) \in Bimod(F_2)$, then $\alpha \notin F_2(X)$ and we arrive at the contradiction. So, $Bimod(F_1) \neq Bimod(F_2)$.□

According to Corollary (2.4) there exists an one-to-one correspondence between classes $Bimod(C)$ and structural monotonic operations. This result is closely parallel to the respective characterization of structural consequences by means of logical matrices. One can develop a theory of bi-matrices parallel to the respective results on logical consequence. We leave this task for another paper and concentrate in the rest of this paper on the properties of bi-matrices which are important for investigating presuppositions.

Let $M = (A_1, D_1, E_1)$ and $N = (A_2, D_2, E_2)$ be similar bi-matrices. The bi-matrix M will be called a *sub-bi-matrix* of the bi-matrix N, in symbols $M \subseteq N$, if A_1 is a subalgebra of the algebra A_2 and also $D_1 = A_1 \cap D_2$, $E_1 = A_1 \cap E_2$.

Similarly as 1.3 a) one can prove the following:

THEOREM 2.5. *Let M, N be bi-matrices for a language S. If $M \in S(N)$, then $Fn_N \leq Fn_M$.*

3 Presuppositions.

Let us consider the following definition of presupposition which mirrors the idea presented by Strawson ([49] pp. 175 - 176):

(1) P presupposes Q if and only if Q is true provided P is true or P is false.

According to (1) Q has to be true in order for P be true or false. Assuming also the bivalence principle (5) we obtain that Q has to be true independently of the logical value of P, hence Q is a tautology. Below we will present this argument in a formal way. That fact is well known. Levinson [83] pp. 175 -176 presented it in detail and then states that: "It has been shown that perfectly well-behaved logic with three values can be constructed and it could be claimed that such a logical systems are (by virtue of their ability to handle presupposition) a notable advance in models of natural language semantics." We are going to show that Levinson is mistaken. Introducing a third logical value does not save the Strawson definition. Surprisingly, it appears that also without assuming (5) the only presuppositions admitted by (1) are classical tautologies.

Strawson's definition formulated in (1) makes the notion of presupposition depend on the notion of logical entailment. We will investigate the relation between logical entailment and presupposition defined by it. Suppose that C is a logic (structural consequence operation). Then the part of (1) consisting of "if P is true then Q is true" can be obviously formulated by means of $Q \in C(P)$. The remaining part "if P is false then Q is true" is more complicated. Any inference with a false premise is valid. For this reason,in order to use here the notion of entailment, we have to use the notion of negation. Suppose then, that the language of C contains a negation connective $\neg$ then (1) might be expressed as:

(2) P C-presupposes Q if and only if $Q \in C(P)$ and $Q \in C(\neg P)$.

The minimal assumption on the negation operator $\neg$ is that it is an unary operator in a given language satisfying the condition (3) below. (3) is weaker than the condition (4) satisfied by the classical negation, as well as, among others, by the negation connectives in many-valued logics.

(3) If P is false then $\neg P$ is true.

(4) $\neg P$ is true if and only if P is false.

(5) Any sentence P is either true or false.

The notion of presupposition defined by means of (2) will be called *presupposition via negation.* This notion has the following constraints:

It depends strictly on the logical consequence operation.

It can be applied only for logical consequences with a negation connective satisfying (3)

In general it does not yield a method to find out the presupposition of many sentences.

Strawson's definition (1) can be approached also in other, more general and perhaps more direct way by means of bi-matrices. This approach leads us to general formalization of Strawsonian presupposition. In the previous section we gave an example of the operation of presupposition just as an illustration of technical usefulness of bi-matrices. Here we will discuss this matter in detail.

Let's consider the operator Fn of presupposition in the following intuitive sense: For a given set X of sentences the set $Fn(X)$ consists of all the sentences Q which are presuppositions of all the sentences from the set X. The formalization of (1) in terms of the operator Fn employs three notions: logical entailment and two classical logical values. The same notions are employed in the operation determined by the definition bi-matrix: Let $Bm(\mathbf{2})$ denotes bi-matrix $(\mathbf{2}, \{0,1\}, \{1\})$, where $\mathbf{2}$ is a two-element Boolean algebra. We will call bi-matrix $Bm(\mathbf{2})$ *the classical presuppositional bi-matrix.* The operation $Fn_{Bm(\mathbf{2})}(X)$ determined by $Bm(\mathbf{2})$ formalizes intuitions expressed by (1) with the assumption of (5). Thus $P \in Fn_{Bm(\mathbf{2})}(X)$ if and only if P is true provided all the sentences from X are true or false. Unfortunately this operation contradicts the usual sense of the notion of presupposing, since we have:

THEOREM 3.1. *For any set of sentences X the set $Fn_{Bm(\mathbf{2})}(X)$ is equal to the set of all classical tautologies.*

Proof. Suppose that $P \in Fn_{Bm(\mathbf{2})}(X)$, then for any valuation v such, that $v(X) \subseteq \{0,1\}$ we have $v(P) = 1$. But obviously any valuation satisfies the condition $v(X) \subseteq \{0,1\}$. As a consequence for any valuation v, $v(P) = 1$, and hence P is a classical tautology.

Given any classical tautology P, obviously for any valuation v $v(P) = 1$, of course any valuation satisfies the condition $v(X) \subseteq \{0,1\}$, then $P \in Fn_{Bm(\mathbf{2})}(X)$. □

Theorem 3.1 proves that in classical logic there are no contingent presuppositions. Obviously it contradicts elementary intuitions concerning presuppositions and proves that Strawson's notion of presupposition makes no sense in classical logic. A similar statement formulated in a slightly different formal setting has been proved in Kracht [93]. It is then necessary to reject the bivalence principle (5) and admit that besides of being true or false a given sentence can have some other logical value.

Let us note that (1) uses the notions of truth and falisty but does not exclude that there are some other logical values. It allows us generalize the notion of classical presuppositional bi-matrix.

A bi-matrix $Bm = (A, D, E)$ will be called a *presuppositional bi-matrix* if and only if the classical presuppositional bi-matrix $Bm(\mathbf{2})$ is a sub-bi-matrix of Bm and moreover $D = \{1, 0\}$ and $D = \{1\}$, where 1 and 0 denote respectively unit and zero element of two element Boolean algebra **2**. We suppose a class of presuppositional bi-matrices **K**. By the *presupposition operator* we will mean the operator Fn_K.

By a *Strawsonian presupposition* we mean any presuppossition operator.

We will not discuss here what the status of other logical values is and in what extent it is justified to use the name "logical value" for the third element of the algebra **sk3** or to other elements of given bi-matrix. By the *classical logical value* we mean just 1 and 0 - the truth and the false, while by *logical value* we mean any element of a given bi-matrix. Then classical logical values are logical values but there exist many non-classical logical values.

The original Strawsonian definition of presupposition (Strawson [49] pp. 174-176) explicitly introduces a kind of third logical value into the trichotomy: true, false, meaningless. The idea of interpretation of Strawson's definition in many-valued logics has been then extensively elaborated. We refer the reader to Beaver [97] for a review of three and four-valued logical systems interpreting Strawson's presupposition.

We will now introduce two algebras determining well known three-valued logics. Let **lu3** denote the three valued Łukasiewicz algebra which differs from **sk3** only by the value of $1/2 \to 1/2 = 1$. By **wk3** we mean the following weak Kleene three-valued algebra. Precisely:

wk3

$\vee$	0	1/2	1
0	0	1/2	1
1/2	1/2	1/2	1/2
1	1	1/2	1

$\wedge$	0	1/2	1
0	0	1/2	0
1/2	1/2	1/2	1/2
1	0	1/2	1

$\to$	0	1/2	1	$\neg$
0	1	1/2	1	1
1/2	1/2	1/2	1/2	1/2
1	0	1/2	1	0

lu3

$\vee$	0	1/2	1
0	0	1/2	1
1/2	1/2	1/2	1
1	1	1	1

$\wedge$	0	1/2	1
0	0	0	0
1/2	0	1/2	1/2
1	0	1/2	1

$\to$	0	1/2	1	$\neg$
0	1	1	1	1
1/2	1/2	1	1	1/2
1	0	1/2	1	0

For $A \in \{\mathbf{sk3}, \mathbf{wk3}, \mathbf{lu3}\}$ by $Bm(A)$ we mean the bi-matrix $(A, \{1, 0\}, \{1\})$, where 1 and 0 denote respectively the unit and zero elements of two element Boolean algebra $\mathbf{2}$.

The following theorem shows that the third logical value does not improve the Strawson's definition.

THEOREM 3.2. *Let $Bm \in \{Bm(\mathbf{sk3}), Bm(\mathbf{wk3}), Bm(\mathbf{lu3})\}$ then:*
a) Fn_{Bm} is a presupposition operator.
b) For any set of sentences X the set $Fn_{Bm}(X)$ consists of classical tautologies).

Proof. a) Obviously, both **lu3** and **wk3** contain two-element Boolean algebra **2** as a subalgebra. Hence $Bm(\mathbf{2})$ is a sub-bi-matrix of $Bm(\mathbf{lu3})$ as well as of $Bm(\mathbf{wk3})$ and of $Fn_{Bm(\mathbf{lu3})}(\emptyset)$.

We will prove b) for $Bm(\mathbf{sk3})$. The proof for the remaining two algebras is similar.

It is easy to check that $Bm(\mathbf{2})$ is a sub-bi-matrix of $Bm(\mathbf{sk3})$. Hence from (2.5 a) we deduce that for any sentence P and any set of sentences X, if $P \in Fn_{Bm(\mathbf{sk3})}(X)$, then $P \in Fn_{Bm(\mathbf{2})}(X)$. Then from theorem (3.1) we deduce that P is a classical tautology.$\square$

Lets note that $Fn_{Bm}(X)$ usually does not contain all classical tautologies. In fact in some case it is empty. The following theorem describes some properties of $Fn_{Bm}(X)$. Its easy proof is left to the reader.

THEOREM 3.3. Let $Bm \in \{Bm(\mathbf{sk3}), Bm(\mathbf{wk3}), Bm(\mathbf{lu3})\}$. *Then:*
a) $Fn_{Bm(\mathbf{sk3})}(\emptyset) = Fn_{Bm(\mathbf{sw3})}(\emptyset) = \emptyset$.
b) $Fn_{Bm(\mathbf{lu3})}(\emptyset)$ consist of all tautologies of three valued Łukasiewicz logic L.
c) If T is the set of all classical tautologies then for $X \subseteq T$ $Fn_{Bm}(X) = T$.
d) For any set X $Fn_{Bm}(X) = \{\emptyset, L, T\}$. $\square$

The sets of tautologies of the strong Kleene three-valued logic and the weak Kleene three-valued logic are empty. As a consequence (3.3) shows that the set $Fn_{Bm}(X)$ varies between the set of respective three-valued tautologies and the set of classical tautologies.

Theorems (3.2) and (3.3) show that three-valued logics does not work better for the formalization of presupposition than classical logic does on the contrary, in a sense, it work even worse than classical logic. Classical logic does not give us a good tool for formalization of presupposition not because any tautology is a presupposition of any sentence – this is in principle acceptable – but because no contingent sentence can be a presupposition. It appears that other three-valued logics lead us to narrower class of presuppositions of a given sentence than classical logic does.

The following theorem generalizes Theorem (3.2).

THEOREM 3.4. *Let* $\mathbf{K}$ *be a class of bi-matrices such that* $Bm(\mathbf{2})$ *is sub-bi-matrix of some bi-matrix from the class* $\mathbf{K}$*. Then for any set of sentences* X *any element of* $Fn_{\mathbf{K}}(X)$ *is a classical tautology.*

Proof. As $Bm(\mathbf{2})$ is a sub-bi-matrix of some bi-matrix from $\mathbf{K}$, then from (2.5 a) we deduce that for any sentence P and any set of sentences X, if $P \in Fn_{\mathbf{K}}(X)$, then $P \in Fn_{Bm(\mathbf{2})}(X)$. Then from theorem (3.1) we deduce that P is a classical tautology. $\square$

COROLLARY 3.5. *For any presupposition operation* Fn *and any set of sentences* X *each element of* $Fn_{\mathbf{K}}(X)$ *is a classical tautology.*

Theorem 3.4 is negative. It allows us to exclude a number of large classes of elaborated logical consequences from the set of acceptable candidates for formalization of presupposition by means of Strawson's idea . All the many-valued as well as fuzzy logics satisfy assumption of (3.4) and hence have to be excluded. Also all the logical systems based on lattice semantics, for example intuitionistic and intermediate logic, orthologics, are excluded for the same reason. Many kinds of relevant logics also possess a matrix semantics based on lattices (see Czelakowski [01] pp. 328 - 342). In fact it is hard to imagine a logical system formulated in the language with the connectives $\rightarrow$, $\wedge$, $\vee$ and $\neg$ which does not satisfy the assumption of theorem (3.4). The only such logic is trivial inconsistent logic, which, of course, for other reasons cannot serve as a tool for the formalization of the notion of presupposition.

The rest of this section will be devoted to an investigation of presupposition via negation.

Logical consequences are usually semantically defined not by logical matrices but by means of others structures like, for example, possible worlds. It is true that, for such logics, matrix semantic might be artificial if the elements of a given matrix does not mirror any intuitively clear objects like classical logical values of truth and false. Nevertheless all the results and methods presented in this paper remains valid also for logics which are usually defined, for example, by means of some kind of possible world semantics.

We can consider logical matrices just as a formal tool to investigate logical consequences.

A natural question arises. Perhaps the language of classical sentential logic is to poor to formalize the phenomenon of presupposition. What, if we extend it by adding new operators? We are going to show that also this way does not lead us to proper solution. It appears that the results similar to (3.4) and (3.5) can be proved for any sentential language which is an extension of the classical one, i.e. for any language containing the classical connectives and eventually also the other ones.

Let S_0 denote the language of classical logic, i.e. the language with two binary and one unary connective corresponding respectively to conjunction, disjunction and negation. A sentential language S we will be called an extension of S_0 if and only if among connectives of S there exists two binary and one unary operators.

According to this definition the modal language with connectives necessitation, conjunction, disjunction and negation is an extension of the classical language S_0. More general, any language obtained by adding new connectives to S_0 is an extension of S_0.

Given a sentential language S and a logical consequence operation C in the language S. Suppose that S is an extension of S_0. It is easy to observe that any sentence of S_0 is also a sentence of S. By a *reduct* of C to S_0 we mean the consequence operation C_0 defined on S_0 in the following way: $P \in C_0(X)$ if and only if $P \in S_0$ and $P \in C(X)$.

The following notion is crucial for the remaining part of this section. A consequence operation C in the language S will be called *sub-classical* if and only if the following conditions are satisfied:

a) S is an extension of S_0.

b) for any $P \in S_0$ and $X \subseteq S_0$ if $P \in C(X)$, then $P \in Cn_{M(\mathbf{2})}(X)$, where $M(\mathbf{2})$ denotes the matrix consisting of two-element Boolean algebra $\mathbf{2}$ with its unit element as the only designated element.

The notion of sub-classical logic is very broad. It is hard to find a logic which is not sub-classical. We will show this in the following example and theorems:

EXAMPLE 3.6. a) By the classical sentential logic we mean the consequence operation Cl determined by the class of matrices of the form $(A, \{1\})$, where

A is a Boolean Algebra and 1 is its unit. It is well known that in particular $Cl = Cn_{M(\mathbf{2})}$, hence Cl is sub-classical.

b) By the trivial (inconsistent) logic Tr we mean the consequence operation in the langauge S defined in the following way: $Tr(X) = S$ for any set of sentence X. Obviously Tr is not sub-classical.

THEOREM 3.7. *If a logic C in the language S_0 is weaker than the classical logic Cl, then C is sub-classical.*

Proof. We have $C(X) \subseteq Cl(X) = Cn_{\mathbf{2}}(X)$. $\Box$

We will show that a very broad class of modal logical consequences satisfies the condition of sub-classicality. However, we have to start by introducing some preliminary notions.

By *modal sentential language* we mean the language $S_\Box = (S, \vee, \wedge, \neg, \Box)$ with two binary connectives $\wedge, \vee$ of conjunction and disjunction, and with two unary connectives: negation $\neg$ and necessitation $\Box$. Thus $S_\Box$ is an extension of the classical language S_0.

The notion of a modal system is understood here as widely as possible. A *modal system* is any set of sentences of the language $S_\Box$ containing all classical tautologies and closed under substitution and modus ponens. A modal system L is called *classical* (see Segerberg [1971]), if L is closed under following rule of extensionality:

$RE \qquad P \leftrightarrow Q / \Box P \leftrightarrow \Box Q$

It is easy to observe that any normal modal system is classical, but also many of non-normal modal system are classical. For details we refer to the monograph G. E. Hughes, M. J. Cresswell [96].

The notion of a modal system, in principle, does not equip us with the notion of entailment. We will define two types of modal consequence operation formalizing modal logical entailment.

Suppose that a modal system L is given. By $L^{\rightarrow}$ we mean the consequence operation in the language $S_\Box$ defined in the following way: for any set $X \subseteq S_\Box$ $L^{\rightarrow}(X) \subseteq S_\Box$ is the least set of sentences containing L and X and closed with respect to modus ponens. Thus $L^{\rightarrow}(\emptyset) = L$ is closed under substitution while, in general $L^{\rightarrow}(X)$ does not need to be close under substitution.

In the literature one can also find another notion of modal entailment. Thus, by $L^{\rightarrow}_{\Box}$ we mean the consequence operation in the language $S_\Box$ defined in the following way: for any set $X \subseteq S_\Box$ $L^{\rightarrow}_{\Box}(X) \subseteq S_\Box$ is the least set of sentences containing L and X and closed with respect to modus ponens and the rule of necessitation $P/\Box P$.

An algebra $A = (A, \wedge, \vee, \neg, \Box)$ of type $(2, 2, 1, 1)$ is called a *minimal modal algebra* if $(A, \wedge, \vee, \neg)$ is a Boolean algebra. Let L be a modal system. We say that an minimal modal algebra A is *appropriate* for the system L, if $(A, \wedge, \vee, \neg)$ is a Boolean algebra and A satisfies the equations $P_A = 1$ (P_A is a term over A corresponding to the sentence P) for all $P \in L$.

For example a minimal modal algebra appropriate for the modal system $S4$ satisfies the conditions: $\Box(x \wedge y) = \Box x \wedge \Box y$, $\Box 1 = 1$, $\Box x \leq x$, $x = \Box\neg\Box\neg x$, $\Box x = \Box\Box x$.

The following theorem comes from J.Malinowski [89]:

THEOREM 3.8. *For any classical modal system L, the consequence operations $L_{\Box}^{\rightarrow}$ and $L^{\rightarrow}$ are determined respectively by the following classes $\mathbf{M}_L$ and $\mathbf{N}_L$, where*
$\mathbf{M}_L$ is the class of all matrices of the form $(A, \{1\})$, where A is appropriate for L and 1 is unit element of A.
$\mathbf{N}_L$ is the class of all matrices of the form (A, D), where A is appropriate for L and $D \subseteq A$ satisfies the following conditions:

a) for any $a, b \in D$ and $c \in A$ $a \wedge b, a \vee c \in D$;

b) for any set $E \subseteq D$ if all elements of E satisfy the condition, if $a \leftrightarrow b \in E$ then $\Box a \leftrightarrow \Box b \in E$, then $E = \{1\}$.$\Box$

THEOREM 3.9. *For any classical modal system L the consequence operations $L_{\Box}^{\rightarrow}$ and $L^{\rightarrow}$ are sub-classical.*

Proof. Let $\mathbf{2}_{\Box}$ denotes two element minimal modal algebra. Two-element Boolean algebra $\mathbf{2}$ is a reduct of $\mathbf{2}_{\Box}$ to Boolean operations. It is easy to check that for any $P \in S_0$ and $X \subseteq S_0$ if $P \in Cn_{(\mathbf{2}_{\Box},\{1\})}(X)$, then $P \in Cn_{(\mathbf{2},\{1\})}(X)$. Observe that the matrix consisting of $\mathbf{2}_{\Box}$ and its unit element as a set of designated elements belong to $\mathbf{M}_L$ as well as to $\mathbf{N}_L$.

Suppose a classical modal system L, and suppose that $P \in S_0$ and $X \subseteq S_0$ and $P \in L_{\Box}^{\rightarrow}(X)$ By 3.6 $P \in Cn_{\mathbf{M}_L}(X) \subseteq Cn_{(\mathbf{2}_{\Box},\{1\})}(X) \subseteq Cn_{(\mathbf{2},\{1\})}(X)$. As a consequence $L_{\Box}^{\rightarrow}(X)$ is sub-classical.$\Box$

It is also clear that all the classical tense logic, classical deontic logics and their combinations are sub-classical. More general, any extension of classical logic is sub-classical. It would be a very interesting problem to find a reasonable logic which is not sub-classical.

The main result concerning presupposition via negation is:

THEOREM 3.9. *Given a sub-classical logic C and the sentence $P, Q \in S_0$. Then, if P C-presupposes Q then Q is a classical tautology.*

Proof. Suppose that P C-presupposes Q, then by the definition (2) we have $Q \in C(P)$ and $Q \in C(\neg P)$. By the definition of sub-classical logic we have that $Q \in Cn_{M(\mathbf{2})}(P)$ and $Q \in Cn_{M(\mathbf{2})}(\neg P)$ then for any valuation v such, that $v(P) = 0$ we have $v(Q) = 1$ as well as for any valuation w such, that $w(P) = 1$ we have $v(Q) = 1$. As a consequence $v(Q) = 1$ for any valuation v. □

There is a clear interconnection between Strawsonian presupposition and the presupposition via negation. Given a Strawsonian presupposition operator $Pres$ defined by means of the class of presuppositional bi-matrices K. Then the class of logical matrices $K' = \{(A, E) : (A, D, E) \in K\}$ determines some consequence operation C, and hence by means of (2) it defines presupposition via negation. It is easy to check that both the operators determine the same presuppositions for those sets of sentences for which both of them are defined.

In same sense the Strawsonian presupposition $Pres$ generalizes the presupposition via negation. (2) make senses only for considering presuppositions of a single sentence. In general, there is no clear method of generalizing (2) for presuppositions of the sets of sentences. This results from the use of negation operation. If we do not indicate the concrete logical entailment, we are unable to determine what is the negation of the set of sentences X even if X is finite. The case of an infinite set X causes even more problems. If the logical consequence in (2) satisfies de Morgan laws, then we could identify the negation of (finite) set X with the disjunction of the negations of its elements. However this identification depends strictly on the underlying consequence operation.

The results presented in this paper can be extended for Kartunnen's approach to presupposition (Karttunen [71] (see also Levinson [83] p. 202, J.Martin [77] and [70]). According to it P presupposes Q if and only if possibility of P entails Q and the possibility of $\neg P$ entails Q. We leave the presentation of these result to other paper.

Conclusion. The theorems proved in this paper shows that the literal formalization of Strawson notion of presupposition, based on the logical entailment, does not lead us to any reasonable formal operator of presupposition. However it does not mean that the Strawson idea of presupposition makes no sense. It seems that the problem consists in the misuse of the notion of logical entailment. Let us note that even in the classical Fregean example, the sentence "Kepler died in misery" does not logically entail the sentence "Kepler existed". In the examples presented by Strawson, sentences also do not logically entail their presupposition. It seems that a link between sentences and their presuppositions cannot be determined via a logic. Strawson seems to be conscious of it in the following comment on historical approaches to presupposition (Strawson [50]):

> "Neither Aristotelian nor Russelian rules give the exact logic of any expressions in ordinary langauge, for ordinary language has no exact logic".

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