Presupposition and atomic modification

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Purpose:
Why presupposition triggers induce presuppositions? What do they have all in common ?

Answer: They denote atomic modifiers which "choose" specific elements (atoms, co-atoms or unit elements) from their arguments. Consequently the relation of presupposition is due to the "algebraic necessity" which corresponds to the fact that some specific elements of an atomic Boolean algebra stand in a relation with other elemen ts by "algebraic necessity". For instance atoms entail in virtue of algebraic necessity (by definition) the elements which contain them (and the dual property). Similarly, any element of a Boolean algebra entails its unit element.

Variety and heterogeneity of presupposition triggers
Aspectual and state change verbs (*begin, finish, loose, become*,etc., focus particles, temporal focus particles (again, yet, already..), cleft constructions, some quantifiers (vague quantifiers, inclusion and exclusion quantifiers: *no... except Leo, most... including Lea,*,etc. (Zuber 1998), lexical presuppositions (*orphan, bold, buxom, to win*), "evaluative" (*deprive, boast, criticise*)

Factive verbs: Emotive and non-emotive factives (Emotive factive presuppose knowledge)

Formal framework: Boolean semantics: (1) syntactically and (2) semantically major grammatical categories are Boolean.
(1) It is possible to form complex expression in any major category by application of Boolean connectives, (2) For any major category there exists a corresponding Boolean algebra of possible denotations of that category (denotational algebras; D_{NP}, D_{Det}, D_{Adv}, etc.

Atomicity Denotational algebras are atomic. Atoms of $D_{A/B}$, when there are no constraints of functions from D_B onto D_A are functions $at_{\alpha,a}$ (for α an atom of D_A and $a \in D_A$) such that $at_{\alpha,a}(x) = \alpha$ if $x = a$ and $at_{\alpha,a}(x) = 0_{D_A}$ otherwise.

Examples: (1) NPs denote functions from properties onto truth values. So their denotations are elements of D_{NP} which are sets of sets. Thus for any property P, the function f_P defined as $f_P(X) = 1$ if $X = P$ and $f_P(X) = O$ if $X \neq P$ is an atom of D_{NP}. So atoms of D_{NP} are singletons which contain a set as their unique element. For instance *Only* plays this role

(2) The algebra of generalised cardinals or $GCARD$. By definition $F \in GCARD$ iff for all properties X, Y_1, Y_2 if $card(X \cap Y_1) = card(X \cap Y_2)$ then $F(X)(Y_1) = F(X)(Y_2)$. This algebra is a proper sub-algebra of $CONS$ and contains as a proper sub-algebra the algebra $CARD$. The algebra $GCARD$ is atomic and its atoms are determined as follows: for any property A and any cardinal n such that $n \leq card(A)$, the function $at_{A,n}$ such that $at_{A,n}(X) = \emptyset$ if $X \neq A$ and if not, $at_{A,n}(X) = \{Y : card(X \cap Y) = n\}$ is an atom of $GCARD$. Elements of $GCARD$ are related to the definite article *the*: expressions *the n* denote atoms of $GCARD$.

Modifiers: A modifier is a functional expression of category C/C for various choices of C. The set $RESTR(C)$ of *restrictive functions*, $f_c \in D_C$ satisfying the condition $f_c \leq id_c$, or equivalently, the set of functions satisfying the condition $f_c(x) \leq x$, for any $x \in D_C$. A sub-algebra $ABS(B)$ of restrictive functions: By definition $f \in ABS(B)$ iff for any $x \in B$, we have $f(x) = x \cap f(1_B)$. We have the following properties:

Prop 2: Let B be a Boolean algebra. Then the set of functions f from B onto B satisfying the condition $f(x) \leq x$ forms a Boolean algebra R_B with the Boolean operations of meet and join defined pointwise: $0_{R_B} = 0_B$, $1_{R_B} = id_B$, $f'(x) = x \cap (f(x))'$
Prop 3: If B is atomic so is R_B. For all $b \in B$ and all atoms α of B such that $\alpha \leq b$, functions $f_{b,\alpha}$ defined by $f_{b,\alpha}(x) = \alpha$ if $x = b$ and $f_{b,\alpha}(x) = O_B$ if $x \neq b$ are the atoms
Prop 4: If B is atomic so is $ABS(B)$. For all atoms α of B functions f_α defined by $f_\alpha(x) = \alpha$ if $\alpha \subseteq x$ and $f_\alpha(x) = O_B$ otherwise, are the atoms of $ABS(B)$

Important: atoms of functions denoted by functional expressions of category C/D are always determined by, or are "indexed" by, the atoms of the denotational algebra of the resulting expression of category C. Very often atomic functions are double indexed: by an element of D_D and by an element of D_C. Thus non-trivial values of atomic functions are determined by the index which depends on the possible argument. For instance *only* in *only Leo* is different from *only* in *only Lea*

Proposal: Presupposition triggers denote atomic (or co-atomic) restrictive functions (determined by their argument). Conventional (lexical) presuppositions correspond to unit elements of restrictive algebras. Entailments of presuppositions are presuppositions.

Examples: Atoms are exceptional elements in the sense that they are not entailed by any (non trivial) element.

(1) *the n* denotes the atom $at_{A,n}$ of the $GCARD$.
(2) *also* is a negation of *only*
(3) exclusion and inclusion determiners are related by the negation (and exclusion determiners denote atomic functions)
(4) *to know that* and aspectual verbs denote atomic functions. Other non-emotive factive verbs are related to *know* (via negation and focus particles); *remember*= still know, forget=not know anymore)
(5) "professional CN" have presuppositions determined by the unit elements of different restrictive algebras (*HUMAN, ANIMATE*, etc.
(6) Presuppositions of) emotive factives correspond to the unit element *to know*

Consequences:
(1) Not only sentences can presuppose.In particular NPs non-declarative sentences have presuppositions (*Every student except Leo and Lea* presupposes that Lea is a student.
(2) There is a problem of presupposition projection at sub-sentential level (example with *also* which is categorially polyvalent and induced different presuppositions depending on the constituent to which it applies.
(3) Presuppositions can "vanish" because presupposition triggers may cease to denote atomic functions in some contexts (when there is a "category switch"). This is because atoms of a sub-algebra are not ("anymore") atoms of the full algebra.
(4) classical logic is not sufficient to define presuppositions because in the two elements algebra the value TRUTH coincides with the only atom of this algebra.

References

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